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Kronecker Linear Discriminant Analysis for Brain-Computer Interfaces

ERP-based brain-computer interfaces

The EEG signal

The EEG signal is a multi-channel time series measured at fixed intervals with a high temporal resolution. It is cut into time intervals (epochs) which are classified.

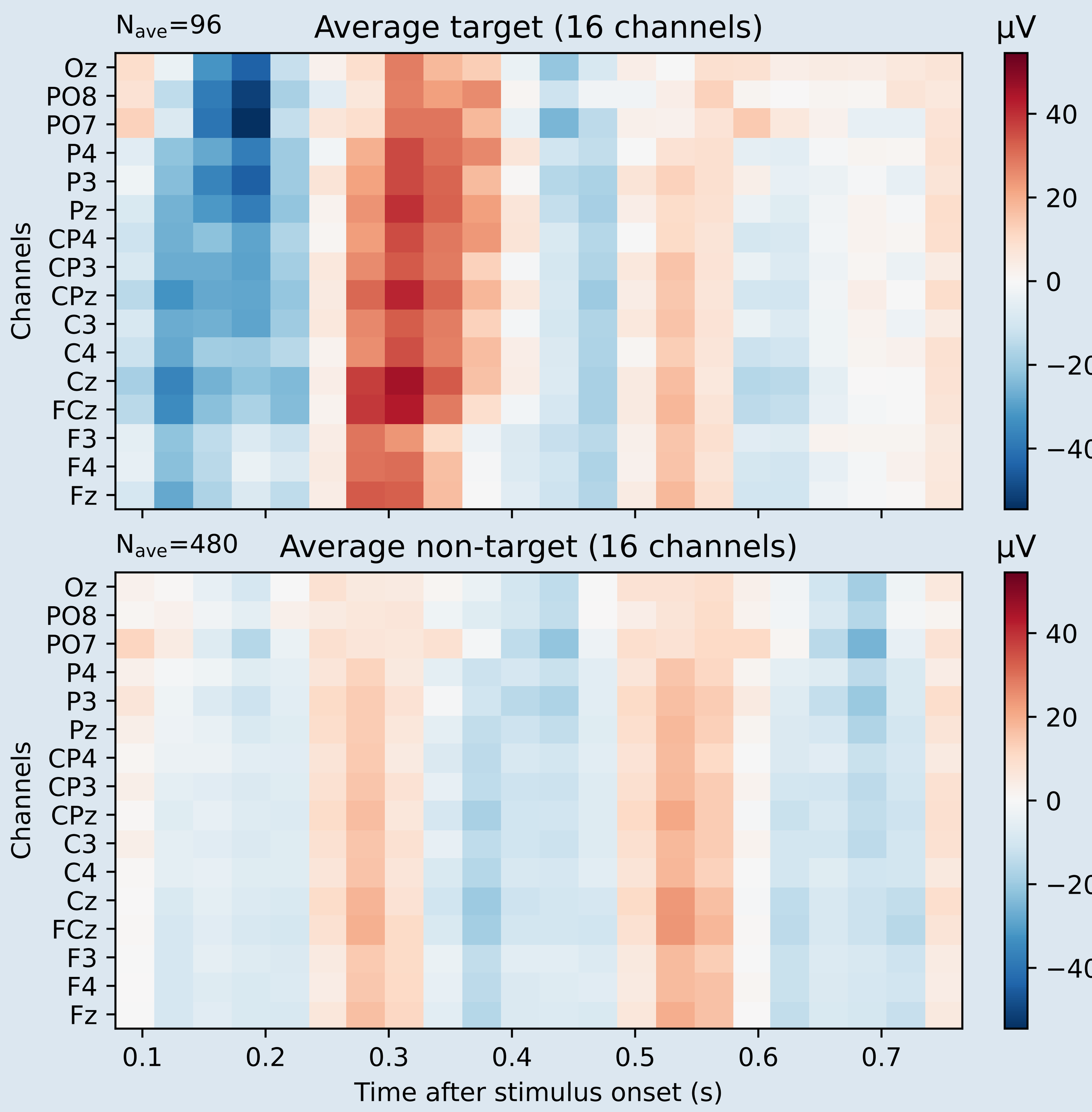
Event-related potentials

Event-related potentials (ERPs) are series of deflections in the EEG signal in response to an attended stimulus. ERPs can be exploited for communication.

Brain-computer interfaces (BCIs) for disabled patients can operate by presenting a user with a series of stimuli and detecting ERPs in the EEG. BCI usability benefits from short calibration time.

High-dimensional LDA for ERP classification

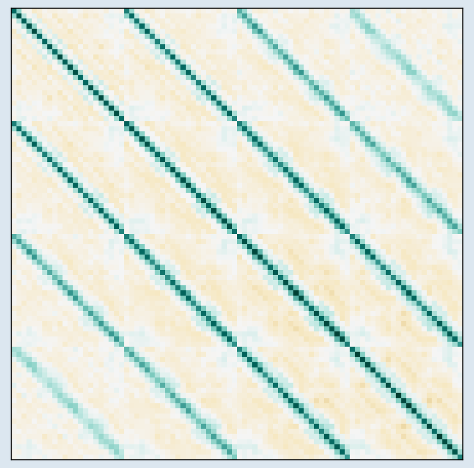
Due to the multi-channel nature and high temporal resolution of the EEG, epochs have a high number of spatiotemporal features. Linear methods have shown successful in classifying ERPs, but often use a selection of hand-picked features and fail to take into account all spatiotemporal features. However, when using all spatiotemporal features it becomes a challenge to apply linear methods like LDA, since the covariance matrix is underdetermined.



Kronecker covariance structure

For epochs $X_i \in \mathbb{R}^{c \times s}$, the covariance $C \in \mathbb{R}^{cs \times cs}$ of the permuted feature vector $x_i = \text{vec}(X_i)$ s.t. $x_i \in \mathbb{R}^{cs}$ can be described as as sum of kronecker products [1].

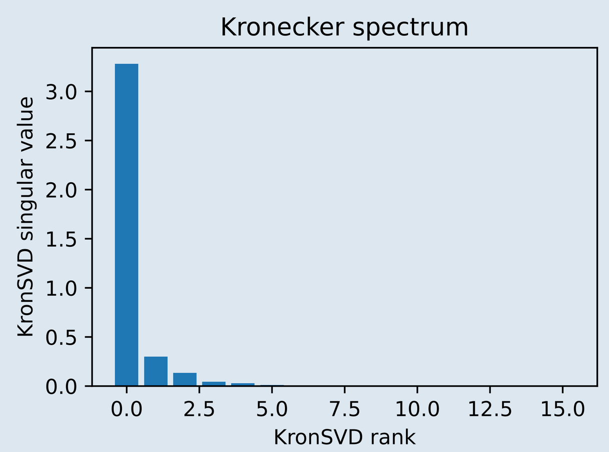
$\hat{C}_{\text{scm}} =$



Sum of KP model

For spatial covariances $S_r \in \mathbb{R}^{c \times c}$ and temporal covariances $T_r \in \mathbb{R}^{s \times s}$:

$$C \approx \sum_r S_r \otimes T_r$$



Single KP model

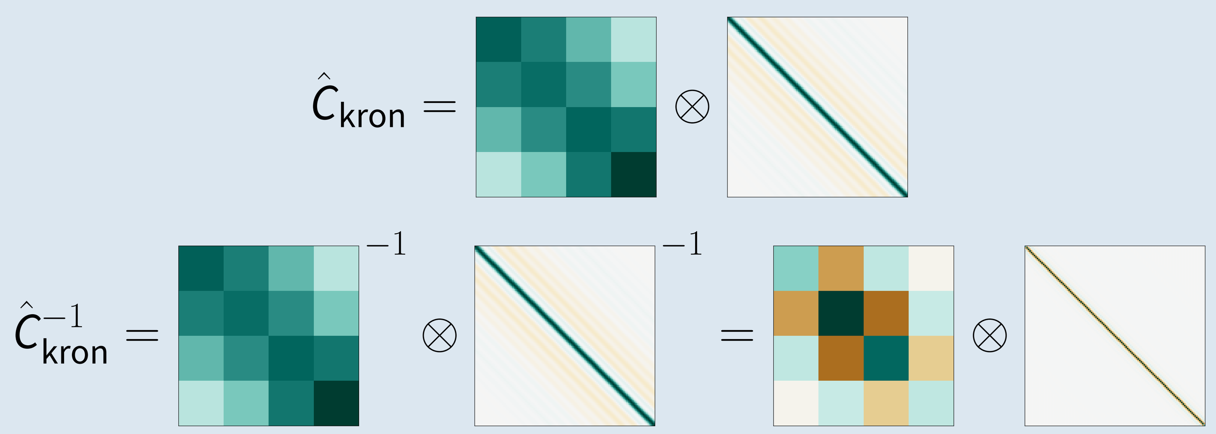
For spatial covariance $S \in \mathbb{R}^{c \times c}$ and temporal covariance $T \in \mathbb{R}^{s \times s}$:

$$C \approx S \otimes T$$

Permuted Kronecker singular value decomposition [2] reveals that spatiotemporal EEG covariance is dominated by a single Kronecker product, hence the single KP model is approximately applicable.

Inverse kronecker covariance

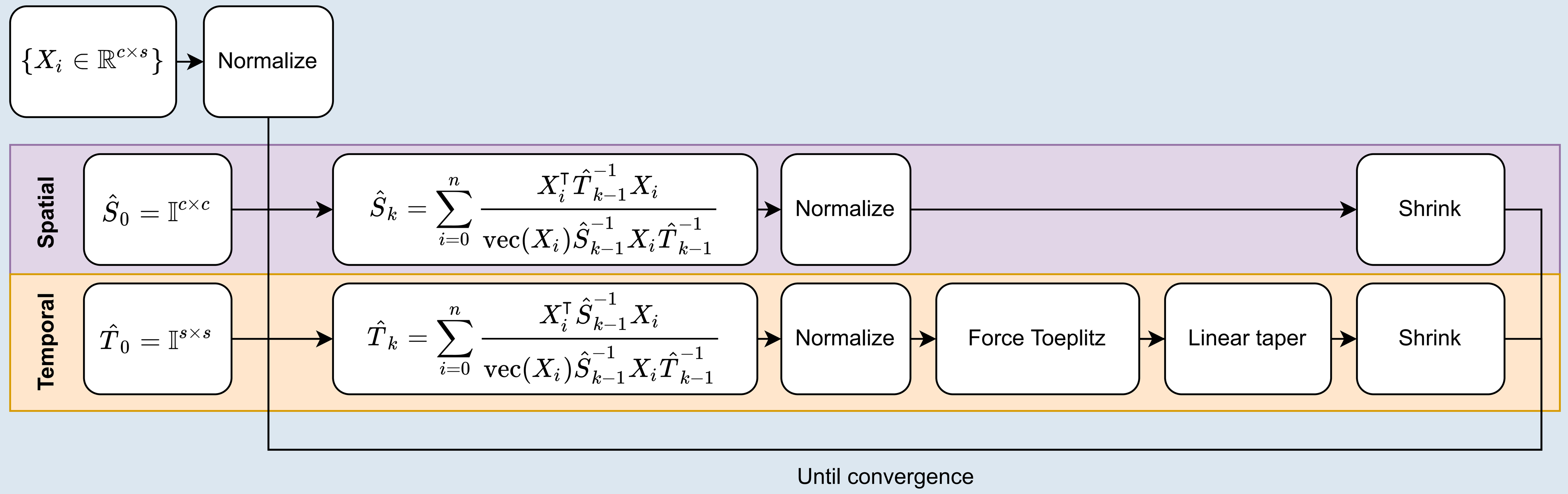
Due to property $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$:



In the Single KP model, there is no need to calculate the full (inverse) covariance. This greatly reduces the number of parameters necessary to calculate the LDA weights, which has a regularizing effect and makes the technique scalable to high resolution data [4].

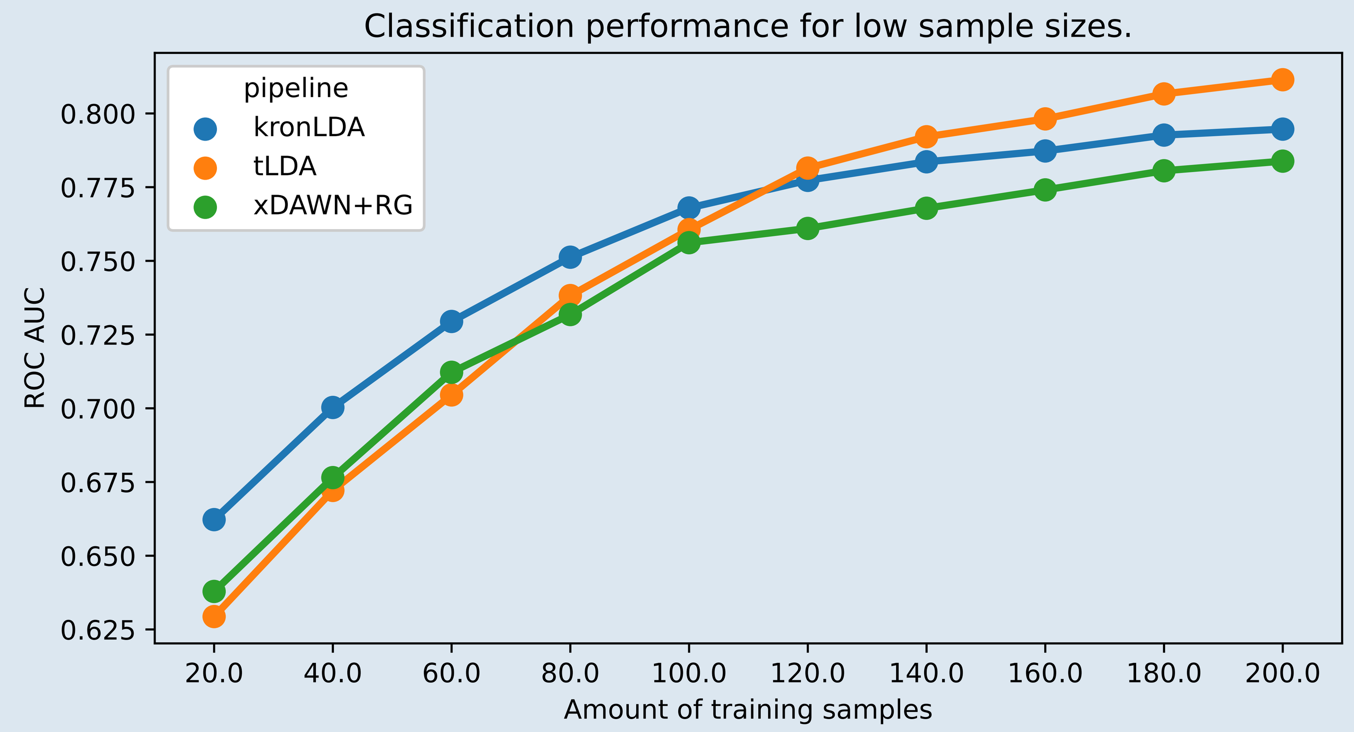
Kronecker covariance estimation algorithm

The spatial and temporal covariance matrices can be efficiently estimated with a fixed point iteration algorithm. Since temporal noise is stationary and measured at fixed intervals, T can be assumed to have Toeplitz structure. Imposing this structure on the temporal covariance estimator has an additional regularizing effect. A linear taper improves Toeplitz inversion stability [3].



Results

Kronecker-LDA is light-weight and improves BCI decoding performance for small training sample sizes, which allows for the shortening of BCI calibration sessions, improving BCI usability.



References

[1] Fetsje Bijma, Jan C. de Munck, and Rob M. Heethaar. "The spatiotemporal MEG covariance matrix modeled as a sum of Kronecker products". en. In: *NeuroImage* 27.2 (Aug. 2005), pp. 402–415.

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[3] Jan Sosulski and Michael Tangermann. "Introducing Block-Toeplitz Covariance Matrices to Remaster Linear Discriminant Analysis for Event-related Potential Brain-computer Interfaces". In: *arXiv:2202.02001 [cs, q-bio]* (Feb. 2022). arXiv: 2202.02001.

[4] Arne Van Den Kerchove et al. "Classification of Event-Related Potentials with Regularized Spatiotemporal LCMV Beamforming". en. In: *Applied Sciences* 12.6 (Jan. 2022), p. 2918.